Is the Peak Value of σ_{xx} at the Quantum Hall Transition Universal?

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The question of the universality of the longitudinal peak conductivity at the integer quantum Hall transition is considered. For this purpose, a system of 2D Dirac fermions with random mass characterised by variance g is proposed as a model which undergoes a quantum Hall transition. Whilst for some specific models the longitudinal peak conductivity σ_{xx} was found to be universal (in agreement with the conjecture of Lee et al. as well as with some numerical work), we find that σ_{xx} is reduced by a factor $(1+g/2\pi)^{-1}$, at least for small g. This provides some theoretical evidence for the non-universality of σ_{xx}, as observed in a number of experiments.

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The integer quantum Hall effect is characterized by the very accurate and robust plateaux of the Hall conductivity $\sigma_{xy} = ne^2/h$, where n is an integer [1]. It is a common belief that the accuracy of the plateaux is a consequence of the fact that the Hall current at the plateaux is carried by edge states [2]. This picture is also supported by the observation that the longitudinal conductivity σ_{xx} vanishes at the Hall plateaux, since this is a dissipative quantity which requires extended bulk states.

At the transition between two Hall plateaux, on the other hand, σ_{xx} becomes nonzero, indicating that extended bulk states are created. It has been conjectured in the literature that the maximum of σ_{xx} has also a universal value in units of e^2/h like σ_{xy}, independent of the specific properties of the sample [3–5]. Specifically, a universal value was found in a number of numerical studies of the lowest Landau level projected electrons [4,6] and of the Chalker–Coddington network model [5]. Similarly, a universal value of $e^2/\pi h$ was found through investigations based on field theory. For instance, Hikami et al. [7] obtained this value in a model for electrons at the lowest Landau level with random spin-scattering. The same value was also found later for Dirac fermions with a random vector potential [8], independently of the strength of randomness, and for Dirac fermions with a weakly-random mass [9]. This is remarkable, because Dirac fermions describe the Hall plateaux as well as the transition between Hall plateaux [8–10].

Experimentally, the value of σ_{xx}^{max} is much less robust than the Hall plateaux and varies between 0.2 and 0.5 in units of e^2/h [11–15]. In particular, in a recent experiment it was found that σ_{xx}^{max} presents for two different samples, at the same low temperature and filling factor, variations as important as 40 % [15].

From a fundamental point of view it appears to be rather unlikely that the dissipative conductivity σ_{xx} is so robust so as to be unaffected by the properties of the material, e.g. by impurities. This problem was discussed recently by Ruzin et al. [16] and Cooper et al. [17] using percolation theory for the random carrier distribution in a disordered sample. They concluded from their calculations that there cannot be a universal conductivity peak height. This raises the question as to whether the models studied in [7,8] describe a generic situation (such as in experiments) or only a very special case in which the effect of disorder is restricted due to special conservation laws. Indeed, it was already discussed in Ref. [8] that a random vector potential is not a generic case since randomness in the vector potential does not create electronic states at the Hall transition. The existence of these states, however, is necessary in order to describe a realistic situation. Therefore, a generic model is probably more like a combination of a random vector potential as well as a random Dirac mass. This view is also supported by the Chalker–Coddington network model which is in the large–scale limit equivalent to Dirac fermions with random vector potential, random Dirac mass and random energy [18].

The simplest model with non-vanishing density of states at the Hall transition is given by 2D Dirac fermions with a random mass. Although this does not represent the most general case, it may be appropriate to investigate the effect of more realistic randomness on the maximum value of σ_{xx} at the Hall transition. For this purpose we will extend the evaluation of σ_{xx}^{max}, performed by one of the authors for a weakly random Dirac mass [9], to stronger randomness.

The 2D Dirac fermion model is defined by the Hamiltonian [8–10]

$$H_D = M\sigma_3 + i\nabla_1\sigma_1 + i\nabla_2\sigma_2, \tag{1}$$

where σ_j are Pauli matrices, ∇_j the gradient in j-direction and M a random mass with mean m and correlation $\langle M_r M_{r'}\rangle = g\delta_{rr'}$. The longitudinal conductivity σ_{xx} can be evaluated from the two–particle Green's function $C(r,\eta) = \sum_{jj'}\langle|(H_D + i\eta)^{-1}_{jj',r0}|^2\rangle$ [8] which connects the origin $r = 0$ of the two–dimensional electron gas with the site r. $\langle ... \rangle$ refers to the average over the random Dirac mass. According to Ref. [9] the Fourier components of $C(r,\eta)$ are given by a function of the 2D wave vector k and the frequency η

$$\tilde{C}(k,\eta) = (\eta'/2g)[\eta + (g\eta' D'/2)k^2]^{-1}. \tag{2}$$

That is, the average two-particle Green's function describes a diffusion process with diffusion coefficient $D = g\eta' D'/2$, where D' is given by

$$D' = 4\alpha\Big[1 + \alpha\big(\frac{\mu^2}{1/g - 2\alpha\mu^2} + \frac{\mu^{*2}}{1/g - 2\alpha\mu^{*2}}\big)\Big] \qquad (3)$$

with $\mu = m' + i\eta'$ and

$$\alpha = \int (|\mu|^2 + k^2)^{-2} d^2k/4\pi^2 \sim \frac{1}{4\pi(m'^2 + \eta'^2)}. \qquad (4)$$

The parameters μ' and η' were evaluated in saddle point approximation [9]. They obey the following equations

$$\eta' - \eta = \eta' g I \quad \text{and } m' = m/(1 + gI) \qquad (5)$$

with $I \sim -\frac{1}{\pi}\ln|\mu|$.

The longitudinal conductivity is directly connected with the diffusion coefficient via the Einstein relation

$$\sigma_{xx} = \frac{e^2}{\hbar} D\rho, \qquad (6)$$

where ρ is the density of states. The latter is given in the model under consideration as $\rho = \eta'/\pi g$, and D' can be approximated in (3) as $D' \approx 4\alpha$ for very weak disorder ($g \approx 0$) [9]. This yields a peak conductivity $\sigma_{xx}^{max} \approx e^2/\pi h$. In the following we will extend this result to the case of stronger disorder by taking the full expression of D' in (3). Although this requires a complicated calculation for the m–dependent conductivity in general, we will find a surprisingly simple result if only the peak value is considered.

In principle we can evaluate the parameters μ' and η' by solving Eq. (5) in order to obtain σ_{xx}. However, this is not necessary if we use the fact that the maximum of σ_{xx} is at $m = 0$ [9], the point where the average mass of the Dirac fermions vanishes. In this case we have $\alpha = 1/4\pi\eta'^2$ and

$$D' = \frac{1}{\pi\eta'^2}\frac{1}{1 + g/2\pi}. \qquad (7)$$

Moreover, for the diffusion coefficient this yields immediately $D = (g\eta'/2)D' = 1/2\pi^2\rho(1 + g/2\pi)$ and eventually for the peak conductivity

$$\sigma_{xx}^{max} = \frac{e^2}{h\pi}\frac{1}{1 + g/2\pi}. \qquad (8)$$

Thus the maximum of σ_{xx}^{max} depends on the strength of the random fluctuations g. The universal value of Dirac fermions with random vector potential agrees only when the variance of the Dirac mass vanishes, $g = 0$, that is for the pure system. For real samples, however, g might be small (e.g., 0.1 in units of the hopping energy of the electrons) such that the reduction of the peak height is small. This is in good agreement with recent experiments which found $\sigma_{xx}^{max} \approx 0.2 \cdots 0.35 e^2/h$ for different samples [15]. For very strong randomness $g \gg 1$ there may be a transition to a Hall insulator [14] and our Dirac fermion model would not be probably sufficient to describe this behaviour adequately.

In conclusion, we have shown that generic randomness in the 2D Dirac fermion model for the integer quantum Hall transition displays no universality in the peak value of the longitudinal conductivity. Our calculation supports the findings by Ruzin et al. [16] and Cooper et al. [17], as well as the various experimental observations in the literature. Moreover, our value of σ_{xx}^{max} is in good agreement with the experimental result of Rokhinson et al. [15].

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